Geometrically adaptive integration over meshfree domains

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We describe adaptive techniques for volumetric and surface integration which are suitable for use by meshfree methods and have been fully implemented in the SAGE meshfree analysis system [1] that is available from http://sal-cnc.me.wisc.edu/Research/meshless. Because geometric domain is not meshed, all integration grids must be generated adaptively and at run time. This naturally suggests use of hierarchical space decompositions of the cells in an initial coarse integration grid that covers the geometric domain.

The most common method is based on the quad/octree space decompositions. Each cell that intersects the boundary of the domain is subdivided into smaller cells; subdivision is terminated when the specified level has been reached. Such space decomposition results in cells that fall into three groups: (1) cells that are completely outside or (2) inside of the geometric domain, and (3) boundary cells — those that intersect the boundary of the domain. Integration over cells belonging to the first two groups is simple: the outside cells are ignored, while integration over the inner cells is performed by sequential application of one-dimensional Gauss integration rules. The integration over the boundary cells reduces to a small number of special cases, similar to those found in the marching cubes algorithm. Depending on how each boundary cell intersects the boundary of the domain, a suitable coordinate parametrization of the boundary is imposed and integration points are allocated according to a Gaussian rule and chosen coordinate parametrization [1].

The hierarchical decomposition allows direct control of the size of the boundary cells, but it does not provide explicit adaptivity to integration errors. For example, not all boundary cells need to be subdivided in order to achieve the desired accuracy of integration. Geometric error over a particular boundary cell may be estimated by integrating the unity function and reduced by proper selection of the primary directions and locations of integration points. Adaptivity to both geometry and integrand requires using the original integrand but requires additional computational effort.

Surface integration requires generating integration points over trimmed parametric surface patches that may be represented by a hierarchical decomposition in the parameter domain. The cells in the decomposition may be rectangular or triangular. In this case, the integrand becomes the composition of the parametric surface equation and the original integrand. The method allows precise control of sampling errors and use of higher-order quadrature rules.

References

[1] I. Tsukanov and V. Shapiro , "The architecture of SAGE - a meshfree system based on RFM", *Engineering with Computers*, v. 18(4), p. 295-311, 2002